

## Open Questions from UnKnot III

### Multi-crossing number and uber-crossing and petal number

1. How do you find these for various knots? Lots of holes in table.
2. Generalize questions about double crossings to triple crossings, quadruple crossings, etc.
3. What is the right notion of unknotting number for triple crossings?
4. What is a crossing change in this setting?
5. What is the corresponding operation for other unknotting operations for triple crossings?
6. Look at bridge number—what if you restrict to triple? Does that change the bridge number? Could it increase?
7. Alternating triple crossing—what should that be? Can alternating theorems generalize? (See papers by Josh in terms of spanning surfaces.)
8. Span of bracket polynomial has relation to crossing number—maybe use result for triple crossing to get at alternating def.
9. How can you construct spanning surfaces with triple/multi crossing projections? Can get checkerboard—spanning surfaces which end up punctured.
10. Connection between hypergraphs and multi-crossing diagrams
11. If you restrict to certain kinds of crossings (double and triple, or a fixed set of numbers) what is minimum number of crossings?

### Mosaic questions

12. What would a triple-crossing mosaic tile look like? (maybe hexagonal tile)
13. Could you have a single 2 by 2 tile with crossing? Quadruple crossings
14. Mosaic tiling with different shaped tiles—octagons for quadruple crossings?
15. Tiling of hyperbolic plane
16. How does Mosaic number behave on composition?
17. Program to calculate invariants based on mosaics: can you find a canonical form?
18. Can you assign a value to the tiles—connection between grid diagrams and mosaics?

19. Use relationship to quantum states and mosaic tiles. Squares in mosaic can be quantum superpositions.
20. 3D mosaic—corresponding mosaic number: similar to question of cubical lattice. Minimize 3D cube that contains the knot versus length of knot.
21. Making torus, sphere, projective plane by gluing edges of mosaic; use fundamental domain to get knots on higher genus surfaces.
22. Mosaic on torus or higher genus—crossings seen across the torus are virtual,
23. Virtual tiles—virtual knots as mosaics
24. Could have paired tiles that represent a handle
25. Games on mosaics—playing crossing-change game on mosaic. Do you always keep things simply connected? May be more interesting on hexagons.
26. Consider Tsuru as a game board to think about. Think about as a tangle.

### **Virtual knots and knotoids**

27. 1,1 tangle: threading is dropping an end through the tangle. What is the unthreading number of a 1,1 tangle? (Rope trick)
28. Does the bracket number detect the unknotoid?
29. Heights of knotoids
30. Unknotting number
31. Slavik Jablan conjecture: start with minimal diagram switch crossings and get another minimal diagram and look for shortest pathway in tree. Does the least pathway down the tree correspond to the unknotting number? Does that conjecture hold for knotoids?
32. Knotoids can be embedded in surfaces as virtual knots of genus one by quotient—show the map is not surjective. Find a knot on the torus that did not come from a knotoid.
33. Conjecture: The knot with Jones polynomial 1 shouldn't be in the image of a knotoid.
34. Count the number of knotoid diagrams with  $n$  crossings. Tabulate knotoids.
35. Any question about knots—multiple crossings, etc, put in this context.

### **Knotty games**

36. Every shadow has some  $k$  for which it is  $k$ -defensible—what are strategies?
37. Playing smoothing game on knotoids
38. Start with  $n$  by  $n$  grid for fixed  $n$ —in mosaic situation—one goal is knot, one for link.
39. Unknotting game: add option to add a Reidemeister move. That would change the number of crossings left.
40. One person tries to create tricolorable, or other favorite invariant. (e.g. make a hyperbolic knot)
41. Choose optimal probability distribution on the crossings for each player—probabilistically determine which crossings to change. Analyze outcomes.
42. Start with crossings on paper. Move consists of joining a pair of crossings and put in an additional crossing on that strand. There will be free ends at end—throw away.
43. Knot cobordism game: could aim for small genus,..
44. Have knot game allowing mutations as a move

### **Physical knots**

45. Non-alternating rope length versus alternating rope length: can you capture property of making rope length of some non-alternating knots acting like lower crossing alternating knots?
46. Closing knots based on some collection of practical knots
47. Relationship between knotoids and physical open knots

### **Legendrian knots**

48. Arrow polynomials for knots with cusps and connection to legendrian knots: cusp knots
49. Fix a knot type. Stabilize enough in mountain range—you should get triple crossings. Does any fixed Legendrian knot have a triple crossing diagram?
50. Fix a smooth type. Which Legendrian representatives have a triple crossing? Minimal triple crossing...
51. Could you do a Legendrian version of uber-number, petal number?
52. Legendrian knotoids—would have a preferred projection

53. Grid diagrams—would Legendrian mosaic number the same thing?

### **Polygons and knots**

54. There is a database of knot diagrams—could you use this to explore Slavik’s conjecture?
55. How many crossings do you need to add to get a diagram with no bigons?
56. Stick number: can this be improved? some torus knots unknown.
57. Equilateral stick number—is there a knot whose equilateral stick number is greater than stick number? Or can you prove they are the same? (hard)
58. What kind of knot-theoretic information can be captured from triangulation of polygons? Is there a relation to known combinatorial information?
59. If you interpolate between knots—if you move from one polygonal projection to another, can you change knot types? Are the regions in convex components?

### **Hyperbolic**

60. Mutants: cusp volume can change and could be further explored.
61. Map ideal lines in upper-half space model back to Euclidean space
62. Mutation and connected to orientation on the knot—some of mutations are orientation reversing. What invariants capture that?
63. Which Klein links are hyperbolic?
64. Mutation on six strands

### **Knots in DNA**

65. Genome: topological complexity. For some random trefoil (or other knot type) what is a method for finding the “trefoiliest” parts of a trefoil?